B.Sc. Part-I Paper I

Page No.

Date: / /

Integral Calculus Notes 2.

Integration by Partial fraction

Let $\frac{F(x)}{\varphi(x)}$ be a rational algebraic function which is to be integrated w.r.t. x. If $\varphi(x)$ can be resolved into linear or quadratic factors then we can change the integrand into algebraic sum of standard integrand by resolving the fraction into partial fractions.

To resolve $\frac{F(x)}{\varphi(x)}$ into partial fractions the degree of $F(x)$ must be less than the degree of $\varphi(x)$. If it is not so, then divide $F(x)$ by $\varphi(x)$ to obtain $\frac{F(x)}{\varphi(x)} = Q(x) + \frac{R(x)}{\varphi(x)}$

where $Q(x)$ is the quotient and $R(x)$ is the remainder whose degree is lower than that of $\varphi(x)$.

The next step is to resolve the denominator into real prime factors. Four such types of factors are considered below.

(1.) Linear non-repeated
(2.) Linear repeated
(3.) Quadratic non-repeated
(4.) Quadratic repeated.

Type I when the denominator of a fraction consists of linear ~~fraction~~ factors (non-repeated) such as $x-a_1$, $x-a_2$ etc. then the given fraction can be expressed as

$$\frac{A}{x-a_1} + \frac{B}{x-a_2} + \text{---}$$ where A, B ---- are constants to be determined and the integration can be performed.

Example.

$$\int \frac{(x-2)\,dx}{(x-1)(x-3)}$$

Sol^n Let $\frac{x-2}{(x-1)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-3)}$

where A and B are constants to be determined

Now $x-2 = A(x-5) + B(x-1)$ —(1)

To obtain A put $x=1$ we get in (1)

we get $-1 = A(-4) + 0$

$$\Rightarrow A = \frac{1}{4}$$

To obtain B, put $x=5$ we get

$$3 = 4B \text{ i.e. } B = \frac{3}{4}$$

$$\therefore \frac{x-2}{(x-1)(x-5)} = \frac{1}{4(x-1)} + \frac{1}{4(x-5)}$$

Hence $\displaystyle\int \frac{x-2}{(x-1)(x-5)} = \frac{1}{4}\int \frac{dx}{x-1} + \frac{3}{4}\int \frac{dx}{x-5}$

$$= \frac{1}{4}\log(x-1) + \frac{3}{4}\log(x-5) + C$$